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PROCEEDINGS

OF

THE ROYAL IRISH ACADEMY.

NOVEMBER 11TH, 1850.

HUMPHREY LLOYD, D. D., PRESIDENT,
in the Chair.

ON the recommendation of the Council,

IT WAS RESOLVED,—That a sum not exceeding £75 be granted for the purpose of defraying the expenses incurred in the erection of the meteorological and tidal instruments.

The President read the following paper on the induction of soft iron, as applied to the determination of the changes of the earth's magnetic force.

"To determine completely the laws of the changes to which the earth's magnetic force is subject, observation must furnish the values of three distinct elements. Of these, the variations in the direction and magnitude of the *horizontal component* are completely determined by the methods given by Gauss and others; but, until lately, no satisfactory means had been devised for the determination of the variations of the remaining element. The principle of the method by which, as I conceive, this desideratum is now supplied, has been already submitted by me to the notice of the Academy;* but as the mode of applying it has since undergone many important alterations, I deem it right to resume the subject, and to lay the method in its complete and amended form before the Society.

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* Proceedings of the Royal Irish Academy, vol. ii. p. 210.

"When a bar of soft iron is held in any direction not perpendicular to that of the earth's magnetic force, it becomes a temporary magnet, by the inducing action of that part of the force which acts in its direction. The *small changes* of the induced magnetism may be assumed to be proportional to those of the inducing force; and, as the former may be measured by their effects, the latter become known.

"To apply this simple principle to the determination of the variations of the *vertical component* of the earth's magnetic force,—two soft iron bars,* of the same size and form, are to be placed vertically, at equal distances on either side of a small freely-suspended horizontal magnet, and so that the plane containing them may pass through the centre of the magnet, and be perpendicular to its axis. Then, if the upper extremity of one of the bars, and the lower extremity of the other, be in (or near) the horizontal plane containing the suspended magnet, it is obvious that they will conspire to deflect it, the predominant pole being in one a north, and in the other a south pole.

"The moment of free magnetism of the suspended magnet being denoted by M, let MU and MU' be the moments of the forces exerted upon it by the two bars. The quantities U and U' are functions of the vertical component of the earth's magnetic force; and depend also upon the quantity and distribution of magnetism in the bars, and upon their position with respect to the suspended magnet. They may likewise each contain a term dependent on the *permanent* magnetism of the bars, which is seldom wholly evanescent. These forces conspire to turn the magnet, and are resisted by the horizontal component of the earth's magnetic force, whose moment is $MX \sin u$,—X denoting the horizontal component, and u the angle of deflection of the magnet from the magnetic meridian. Hence the equation of equilibrium is

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* "The employment of a *second* bar originated with Dr. Lamont, of Munich, to whom (as will presently be stated) this method is indebted also for other improvements."

$$U + U' = X \sin u.$$

"Now let the two components of the earth's force undergo any small changes, δX and δY, and let $V\delta Y$ and $V'\delta Y$ be the changes of U and U' produced by the latter. Then, δu denoting the corresponding change of the angle u, in parts of radius,

$$(V + V')\,\delta Y = X \cos u\,\delta u + \delta X \sin u.$$

Dividing by the equation $Y = X \tan\theta$, in which θ denotes the magnetic inclination, there is

$$(V + V') \tan\theta\,\frac{\delta Y}{Y} = \cos u\,\delta u + \sin u\,\frac{\delta X}{X};$$

or, making, for abridgment, $(V + V') \tan\theta = \frac{1}{p}$,

$$\frac{\delta Y}{Y} = p\left(\cos u\,\delta u + \sin u\,\frac{\delta X}{X}\right).$$

The angle u, in this formula, being the deviation of the suspended magnet from the position which it would assume under the action of the earth alone, its changes, δu, are the differences between the observed changes of position, measured from a fixed line, and the corresponding changes of declination.

"In order to correct for the effect of temperature upon the iron bars, we have only to substitute $(\delta u - a\delta t)$ for δu, δt being the actual change of temperature, and a the change of angle (in parts of radius) corresponding to a change of one degree. The effect of an increase of temperature upon a soft iron bar, in all my experiments, has been an *increase* of its induced magnetism,—the reverse of its effect upon the permanent magnetism of an artificial magnet. The amount of the change is, however, very small. With the bar which has been most used in the Dublin Magnetical Observatory, an increase of 1° Fahr. produces a change of angle amounting only to + 0′·05; so that $a = + \cdot 000015$, and the relative change of the force of the bar = + ·000029.

"If we assume that the induced magnetism of the iron bars is proportional to the inducing force, the coefficient p may be found by *inverting* the bars, and observing the angles of deflection in the direct and inverted positions. For, these angles being denoted by u and u', it may be readily shown that

$$p = \frac{2}{\sin u + \sin u'}.$$

It was by this method that I originally proposed to determine the constant of the preceding formula. The assumption upon which it rests is the same as that which Poisson has taken as the basis of his theory of induced magnetism. It is, however, as Dr. Lamont has shown, not strictly in accordance with fact; and it is therefore necessary to seek another mode of determining the constant. It is obvious that this quantity will be known, if we can alter the inducing force artificially, by a small but known amount, and observe the change of angle thereby produced. This is the principle of the method devised by Dr. Lamont for the purpose; it is practised in the following manner.

"A magnet is placed at a considerable distance above or below the suspended magnet, their centres being in the same vertical line; and it is so arranged as to be capable of rotation round a horizontal axis parallel to the suspended magnet in its deflected position. Let this magnet be first placed *vertically*, in which position it exerts no *direct* action upon the suspended magnet, but only on the iron bars. Then, if R and R' denote the forces exerted by the auxiliary magnet upon the two bars, $\delta U = VR$, $\delta U' = V'R'$; so that if kn denote the corresponding change of angle, expressed in scale-divisions of the instrument, we have

$$VR + V'R' = X \cos u \, kn.$$

Let the deflecting magnet be now turned (the position of its centre remaining unchanged), so that its axis is *horizontal*, and perpendicular to that of the suspended magnet. In this position it exerts no action upon the iron bars; but tends to turn the suspended magnet with a force whose moment we shall denote by S. The equation of equilibrium in this case is therefore

$$U + U' + S = X \sin (u + kn'),$$

kn' being the change of position of the suspended magnet due to the small added force. Hence

$$S = X \cos u \, kn';$$

and, dividing the equation last found by this,

$$V \frac{R}{S} + V' \frac{R'}{S} = \frac{n}{n'}.$$

"Now, the deflecting magnet being vertical, and its distance considerable as compared with its length, the force which it exerts upon the unit of free magnetism at the centre of one of the iron bars, in the direction of the joining line, and in the perpendicular direction, respectively, are

$$\frac{2M}{e^3} \cos \phi, \quad - \frac{M}{e^3} \sin \phi,$$

in which M denotes the magnetic moment of the deflecting magnet, e the length of the line connecting its centre with the centre of the iron bar, and ϕ the angle which that line makes with the vertical.* And the sum of these forces, resolved in the vertical direction, is

$$\frac{M}{e^3} (2 \cos^2 \phi - \sin^2 \phi).$$

But we may consider the quantities e and ϕ (and therefore the force exerted by the magnet) to be the same for all points of the bar, the variations of these quantities being of the same order as those neglected in the approximation; so that

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* Transactions of the Royal Irish Academy, vol. xix. p. 162.

$$R = \frac{M}{e^3}(2 - 3\sin^2\phi), \quad R' = \frac{M}{e'^3}(2 - 3\sin^2\phi');$$

e' and ϕ' denoting the corresponding quantities for the second bar. Also, if a denote the distance between the centres of the deflecting and suspended magnets, we have

$$S = \frac{M}{a^3}.$$

Substituting these values, and observing that $\sin^2\phi = \sin^2\phi'$, very nearly,

$$(2 - 3\sin^2\phi)\left(V\frac{a^3}{e^3} + V'\frac{a^3}{e'^3}\right) = \frac{n}{n'}.$$

"Now, if b denote the horizontal distance of the axis of each bar from the centre of the suspended magnet, and h the distance of their centres above and below the plane in which the latter moves, we have

$$e^2 = (a + h)^2 + b^2, \quad e'^2 = (a - h)^2 + b^2;$$

accordingly, if we expand $a^3 e^{-3}$, $a^3 e'^{-3}$, according to the ascending powers of $\frac{h}{a}$, $\frac{b}{a}$, (stopping at the second), we find

$$V\frac{a^3}{e^3} + V'\frac{a^3}{e'^3} = (V + V')\left(1 + 6\frac{h^2}{a^2} - \tfrac{3}{2}\frac{b^2}{a^2}\right) + 3(V' - V)\frac{h}{a};$$

in which, since V and V' are nearly equal, the term $3(V' - V)\frac{h}{a}$ may be neglected. Also $\sin^2\phi = \frac{b^2}{e^2} = \frac{b^2}{a^2}$, $q.p.$ And, substituting these values in the formula obtained above, it becomes

$$2(V + V')\left(1 + 6\frac{h^2}{a^2} - 3\frac{b^2}{a^2}\right) = \frac{n}{n'}.$$

But $p = (V + V')^{-1}\cot\theta$; wherefore, finally,

$$p = 2\cot\theta\left\{1 + 3\left(\frac{2h^2 - b^2}{a^2}\right)\right\}\frac{n'}{n}.$$

"In my original instrument there was but *one* iron bar; and it was placed in the vertical plane passing through the centre of the suspended magnet, and *perpendicular to the magnetic* *meridian.* This position, I soon perceived, was less advantageous than that described above; and it was accordingly abandoned in all the instruments subsequently constructed for other observatories. In the Dublin Observatory, however, I preferred the continuance of the less perfect arrangement to the interruption of the series of observations consequent upon its alteration; and I have been thus compelled to re-discuss the theory of the instrument, which is rendered much more complicated in this form.

"It was natural to suppose that the moment of the force exerted by the iron bar upon the suspended magnet was of the form $MU\cos u$, U and u denoting as before; so that the equation of equilibrium should be, simply,

$$U = X \tan u.$$

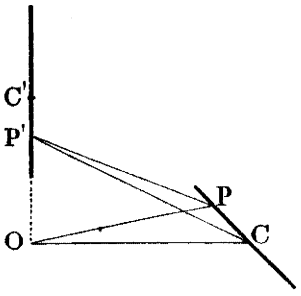
Differentiating, and dividing by $Y = X \tan \theta$, and denoting (as before) the change of U produced by a small change of the vertical component of the earth's force by $V\delta Y$, we have

$$\frac{\delta Y}{Y} = p\left(\sec^2 u\,\delta u + \tan u\,\frac{\delta X}{X}\right),$$

in which $p = V^{-1} \operatorname{cotan} \theta$.

"This formula, however, is only an approximate one. The expression for the moment of the force exerted by the iron bar upon the suspended magnet, in this case, is not, strictly, of the form supposed above; and it becomes necessary to seek the exact form, and to examine in what manner the formula of reduction is modified.

"Let CP be the freely suspended horizontal magnet; C′P′ the vertical iron bar; and O the point in which it intersects the horizontal plane passing through CP. Also, let dm and dm' denote the elements of free magnetism at P and P′, and ρ the distance PP′. Then the force exerted by dm' on dm is



$$\frac{dm\,dm'}{\rho^2};$$

and the portion of this resolved in the horizontal plane, is

$$\frac{dm\,dm'}{\rho^3} \times \mathrm{OP}.$$

The moment of this force to turn the suspended magnet is

$$\mathrm{CP} \times \frac{dm\,dm'}{\rho^3} \times \mathrm{OP} \sin \mathrm{OPC} = \mathrm{CP} \times \frac{dm\,dm'}{\rho^3} \times \mathrm{OC} \sin \mathrm{OCP}.$$

Hence, putting $\mathrm{OC} = a$, $\mathrm{CP} = r$, and $\mathrm{OCP} = 90° - u$, the whole moment of the force of the iron bar is

$$a \cos u \iint \frac{r\,dm\,dm'}{\rho^3}.$$

Now, $\overline{\mathrm{PP}'^2} = \overline{\mathrm{P'O}^2} + \overline{\mathrm{OP}^2}$. Or, putting $\mathrm{OC}' = h$, $\mathrm{CC}' = e$, $\mathrm{C'P}' = r'$,

$$\rho^2 = (h - r')^2 + a^2 + r^2 - 2ar \sin u = e^2 + r^2 + r'^2 - 2\,(hr' + ar \sin u).$$

Accordingly, expanding $\frac{1}{\rho^3}$ according to the ascending powers of $\frac{r}{e}$, $\frac{r'}{e}$, integrating and making

$$\int r^n dm = M_n, \qquad \int r'^n dm' = M'_n,$$

and observing that, on account of the symmetrical distribution of free magnetism in the magnet and bar, M_n and M'_n vanish when n is an even number,—we have, for the moment of the force of the iron bar,

$$\frac{3ah}{e^5} MM' \cos u \Big\{ 1 - \tfrac{5}{2} \frac{1}{e^2} \Big[\frac{M'_3}{M'} \Big(1 - \tfrac{7}{3} \frac{h^2}{e^2} \Big) + \frac{M_3}{M} \Big(1 - 7 \frac{a^2}{e^2} \sin^2 u \Big) \Big]$$
$$+ \tfrac{5 \cdot 7}{2 \cdot 4} \frac{1}{e_4} \Big[\frac{M'_5}{M'} \Big(1 - 6 \frac{h^2}{e^2} \Big) + 2 \frac{M'_3}{M'} \frac{M_3}{M} \Big(1 - 9 \frac{a^2}{e^2} \sin^2 u - 3 \frac{h^2}{e^2} \Big)$$
$$+ \frac{M_5}{M} \Big(1 - 18 \frac{a^2}{e^2} \sin^2 u \Big) \Big] \;\&\text{c.} \Big\}.$$

"This formula is unfortunately not convergent, and is, consequently, of no use in the present investigation. In fact, $\frac{M'_3}{M'}$ is of the same order of magnitude as e^2, $\frac{M'_5}{M'}$ as e^4, and so on. We are, therefore, unable to obtain the value of this moment, expressed as an explicit function of a and h, and must have recourse to a different development.

"Let the distance of any point of the iron bar from the centre of the suspended magnet, CP′, be denoted by R,

$$R^2 = a^2 + (h - r')^2, \text{ and } \rho^2 = R^2 + r^2 - 2ar \sin u.$$

Expanding $\frac{1}{\rho^3}$ according to the inverse powers of R, and integrating, observing that $M_n = 0$, when n is even,

$$\iint \frac{r\,dm\,dm'}{\rho^3} = M \int \frac{dm'}{R^3} - \tfrac{3}{2} M_3 \int \frac{dm'}{R^5} + \tfrac{3\cdot 5}{2\cdot 4}\left(M_5 + 4a^2 \sin^2 u\, M_3\right) \int \frac{dm}{R^7}$$

$$- \tfrac{3\cdot 5\cdot 7}{2\cdot 4\cdot 6}\left(M_7 + 12a^2 \sin^2 u\, M_5\right) \int \frac{dm'}{R^9} + \&c.\,;$$

or, if we make

$$M \int \frac{dm'}{R^3} - \tfrac{3}{2} M_3 \int \frac{dm'}{R^5} + \tfrac{3\cdot 5}{2\cdot 4} M_5 \int \frac{dm'}{R^7} - \&c. = A,$$

$$\tfrac{15}{2}\left(M_3 \int \frac{dm'}{R^7} - \tfrac{7}{2} M_5 \int \frac{dm'}{R^9} + \&c.\right) = B,$$

$$\iint \frac{r\,dm\,dm'}{\rho^3} = A + Ba^2 \sin^2 u\,;$$

in which, on account of the smallness of the distance of the iron bar, the term containing $\sin^2 u$ may bear a very sensible proportion to the whole. Accordingly, if we put, for abridgment,

$$Aa = MU, \qquad Ba^3 = MUQ,$$

the moment of the force exerted by the iron bar is

$$MU \cos u\,(1 + Q \sin^2 u)\,;$$

and the equation of equilibrium therefore is

$$U(1 + Q \sin^2 u) = X \tan u.$$

"Let $V\delta Y$ denote, as before, the change of U produced by a small change of the earth's vertical force. Then, if we write the equation of equilibrium under the abbreviated form $U = Xfu$, and differentiate, and divide by $Y = X \tan \theta$,

$$V \tan \theta \frac{\delta Y}{Y} = f'u \, \delta u + fu \frac{\delta X}{X}.$$

But $fu = \frac{\tan u}{1 + Q \sin^2 u}$; and $f'u = \frac{1 - Q \sin^2 u}{\cos^2 u \, (1 + Q \sin^2 u)^2}$, neglecting the term $2 \, Q \sin^4 u$ in the numerator, as inconsiderable; wherefore putting, for abridgment,

$$p = \frac{\operatorname{cotan} \theta}{V(1 + Q \sin^2 u)}, \quad S = \frac{1}{\cos^2 u} \cdot \frac{1 - Q \sin^2 u}{1 + Q \sin^2 u},$$

there is finally,

$$\frac{\delta Y}{Y} = p \left(S \, \delta u + \tan u \frac{\delta X}{X} \right).$$

" The coefficient p, in this formula, is obtained by the method already explained. It has been shown that when a magnet is placed vertically, above or below the suspended magnet, its inducing action on the iron bar $= \frac{M}{e^3} (2 - 3 \sin^2 \phi)$, e and ϕ denoting as before; and, as this force has the same effect as a small change of the earth's vertical force, the effect upon the suspended magnet is obtained by making $\delta Y = \frac{M}{e^3} (2 - 3 \sin^2 \phi)$, $\delta X = 0$, in the preceding equation. Wherefore, kn denoting the corresponding change of angle,

$$\frac{M}{e^3} (2 - 3 \sin^2 \phi) = p \, YS \, kn.$$

Again, when the deflecting magnet is horizontal, and perpendicular to the magnetic meridian, its effect is the same as that produced by a small change of the earth's horizontal force, whose magnitude is given by the equation

$$\delta X \sin u + \frac{M}{a^3} \cos u = 0.$$

Putting, therefore, for δX the value thus given, and making $\delta Y = 0$,

$$\frac{M}{a^3} = XS \, kn';$$

kn' denoting the corresponding value of δu. And eliminating M between this and the former equation, we find

$$p = \cot\text{an}\,\theta\,(2 - 3\sin^2\phi)\,\frac{a^3}{e^3}\,\frac{n'}{n}.$$

"We must likewise have recourse to experiment, to determine the value of the coefficient S.[*] In fact we have seen that the quantity, Q, which enters into the expression of this coefficient, is the ratio of two series containing the integrals $\int r^3 dm$, $\int r^5 dm$, $\int r^7 dm$, &c., $\int \frac{dm'}{R^3}$, $\int \frac{dm'}{R^5}$, $\int \frac{dm'}{R^7}$, &c., the values of which, depending upon the distribution of free magnetism in the magnet and iron bar, cannot be known *a priori*.[†] We may, however, determine the value of the coefficient S by experimental means analogous to those already employed in the determination of p. We have seen, in fact, that when the deflecting magnet in that experiment was horizontal, and perpendicular to the magnetic meridian, there was

$$\frac{M}{a^3} = XS\,kn'.$$

Now, let the iron bar be removed, and, the deflecting magnet remaining in the same position, let kn'' denote the change of angle produced by its action. Then

$$\frac{M}{a^3} = Xkn'';$$

and, dividing the equation last found by this,

$$S = \frac{n''}{n'}."$$

The President exhibited to the Academy a map of Ire-

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[*] "It is obvious that this necessity does not arise in the adjustment of the soft iron bars described in the commencement of this Paper."

[†] "We may approximate to these values, and therefore to the value of Q, on the assumption that the whole forces of the magnet and bar are concentred in two points, or poles."